An experimental investigation of temperature distribution and heat transfer through a closed rectangular enclosure bounded with solid wall comparing with finite calculation

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Abstract

This research study an experimental and numerical investigation on the temperature distribution in a rectangular enclosure bounded with solid wall. This research have developed the computer program in order to determine the temperature profile in the closed enclosure by using finite different method. The research parameter can be classified to external factor (solar radiation, sky radiation and environmental convection heat transfer) and internal factor(conduction heat transfer , internal convection heat transfer and internal wall radiation). The data collection from the experiment has been collected every 5 minutes 24 hours a day for 1 week. The comparison between computer program and the experimental data have been analyzed by using statistical method. The result found that the correlation coefficient between experiment and calculation during day time is 94% and during night time is 89%

Keywords: Temperature Profile, Heat Transfer , Finite Different, Form Factor , Solar Radiation

1. Introduction

The closed rectangular enclosure is widely used in many of solar energy appliance like box type solar cookers or solar heating equipment [1,4]. There are many of developed mathematical models to predict the heat transfer through the enclosure. Therefore many of researcher has concentrated on the experimentation together with the mathematics modeling [2,3]. The heat transfer characteristics inside the closed rectangular enclosure are normally divided into 3 major components, conduction heat transfer, convection heat transfer and radiation heat transfer [3,8]. The radiation heat transfer components of the closed enclosure are normally investigated and studied as radiant cooling application [5] Therefore this study emphasized on the radiation heat transfer component that effected on temperature distribution profile inside the closed enclosure by combining the technique of radiation heat transfer that have been used in radiant cooling application. This study applied the finite different technique together with the Newton raphson calculation method [9] to solve the temperature profile in the closed enclosure.

2. System Description and Analysis

2.1 System

The system of this study consists of three dimensional rectangular enclosure with the glass top as shown in Fig 1. The box dimension is 0.6 x 0.4 x 0.2 m. The 4 outside walls are painted black to minimize the emission and maximize the absorption [7].

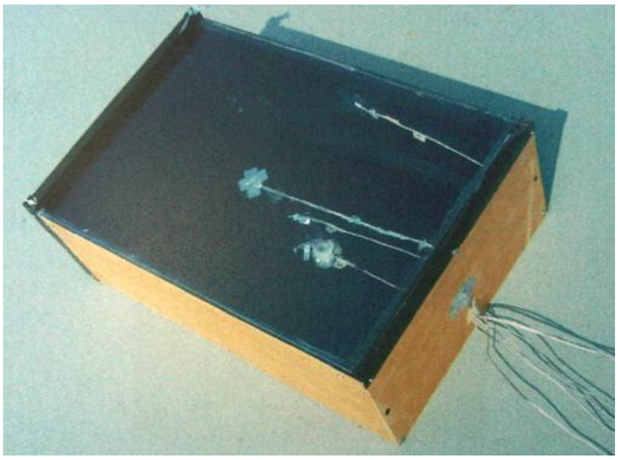

Fig.1 The close rectangular enclosure with set up measuring equipment.

2.2 An experiment setup and methodology

The experimental equipment has been conducted at Energy Park, Asian Institute of Technology , Pathumtani , Thailand (Latitude 13.5 , Longitude 101.25) during Month of January for 7 days . There are 6 points of temperature measurement using Type K thermocouple installed at Glass cover, ground , air inside, inner east side, inner south side and external south side position and using Pyranometer , Thermometer , Anemometer, Pyrgeometer to measure the environment.

3. Solar positioning calculation

The position of the sun at any point on the surface of the earth can be determined in the function of altitude and azimuth angle. The altitude angle, which is in the function of latitude, declination angle, solar hour angle , is the angle on a vertical plane between the sun's direct ray and the projection of this line on the horizontal plane. The azimuth angle ,which is in the function of hour angle, declination angle ,and altitude angle, is the angle on the horizontal plane measured from south to the horizontal projection of the sun's ray.

Equation of Time : The equation of time gives the different between solar time and clock time due to :

$$E_t = 9.87 \cdot \sin(2 \cdot \beta) - 7.53 \cdot \cos(\beta) - 1.5 \cdot \sin(\beta) \quad \text{Eq.(1)}$$

$$\beta = \frac{360}{365} \cdot (J_d - 81) \quad Eq.(2)$$

Solar time(t) and *Hour angle(ω)* : the Solar time is calculated from standard time by the equation

$$solar\ time = \frac{1}{60} \cdot (60 \cdot Local\ time - 4 \cdot (L_{st} - L_{ac}) + E_t) \quad \text{Eq.(3)}$$

$$\omega = 15 \cdot (t - 12) \quad \text{Eq.(4)}$$

Declination angle(δ) : Declination angle can be defined as the angle formed by he line extending from the center of the sun to the center of the earth and the projection of this line upon the earth's equatorial plane. The declination of the sun varies daily and can be calculated from the formula as follows :

$$\delta = 23.45 \cdot \sin\left(\frac{360}{365} \cdot (284 + J_d)\right) \quad \text{Eq.(5)}$$

Solar altitude angle(α) can be determined by following equation,

$$\alpha = \sin^{-1}(\sin(\phi) \cdot \sin(\delta) + \cos(\phi) \cdot \cos(\delta) \cdot \cos(\omega)) \quad \text{Eq.(6)}$$

and *Solar azimuth angle(γ_s)* can be determined by

$$\gamma_s = \sin^{-1}\left(\frac{\sin(\omega) \cdot \cos(\delta)}{\cos(\alpha)}\right) \quad \text{Eq.(7)}$$

Zenith angle(θ_z) : Zenith angle is the incident angle of the sun direct ray to the vertical plane . it can be determined by,

$$\theta_z = 90 - \alpha \quad \text{Eq.(8)}$$

Incident angle(θ) : the incident angle is the angle between the sun's direct ray to the normal of the plane surface, as shown in Fig. 2. To calculate incident angle, the solar vector and plane vector must be determine

$$Solar\ Vector\ (V_s) = \begin{bmatrix} \sin(\alpha) \\ -\cos(\alpha) * \sin(\gamma_s) \\ -\cos(\alpha) * \cos(\gamma_s) \end{bmatrix} \quad \text{Eq.(9)}$$

and

$$Plane\ Vector\ (V_p) = \begin{bmatrix} \cos(\beta) \\ -\sin(\beta) * \sin(\gamma_p) \\ -\sin(\beta) * \cos(\gamma_p) \end{bmatrix} \quad \text{Eq.(10)}$$

$$Incident\ Angle(\theta) = \cos^{-1}[\vec{V}_s \cdot \vec{V}_p] \quad \text{Eq.(11)}$$

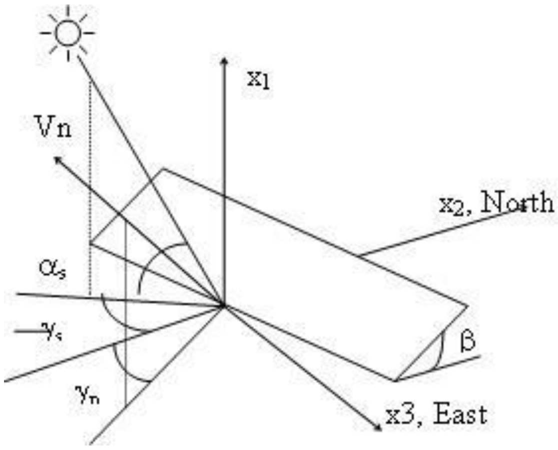

Fig.2 The related angle for calculation of sun direction

4. Energy balance Equation

This study has applied the concepts of finite different method and using the newton raphson method [9] in order to solve the enormous temperature parameter in each node in the considered space. For the calculation of heat gain through wall, the wall node has been set to 3 different nodes, outside position (External node), inside wall position (between layer), and inside position (internal node)

4.1 The External Wall Node

The external node exposed directly to the sun/sky, the heat transfer in external node of the wall can be described as Fig. 3, it can be written the energy balance equation at the outside wall as following,

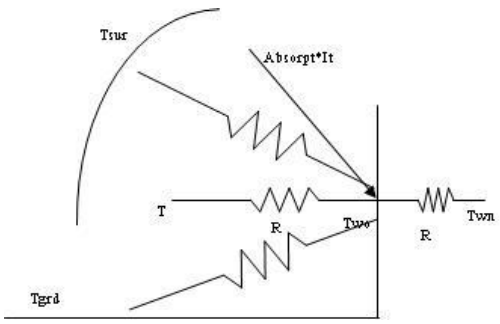

Fig. 3 Heat transfer mechanism at outside wall surface

According to energy balance equation,

$$E_{in}-E_{out} = E_{st} \qquad \text{Eq.(12)}$$

Energy balance of this point is

$$\begin{bmatrix} \alpha_{ab}\cdot I_t + FF_{sur-W}\cdot\varepsilon\cdot\sigma\cdot(T_{surr}^4 - T_{wo}^4) \\ + FF_{grd-W}\cdot\varepsilon\cdot\sigma\cdot(T_{grd}^4 - T_{wo}^4) \\ + h_{co}\cdot(T_o - T_{wo}) \end{bmatrix} = \left[\frac{T_{wo}-T_{wn}}{R_w}\right]$$

Eq.(13)

$$I_t = \quad I_b\cdot\cos(\theta) + 0.5\cdot(1+\cos(\beta))\cdot I_d + 0.5\cdot\rho_g\cdot I_g\cdot(1-\cos(\beta)) \qquad \text{Eq.(14)}$$

4.2 Inside wall node(between layer)

Results in changing temperature in consecutive node, cause conduction heat transfer effect that will effect to the change of temperature in considered node. Fig. 4 illustrates analysis of conduction heat transfer mechanism for inside wall layer.

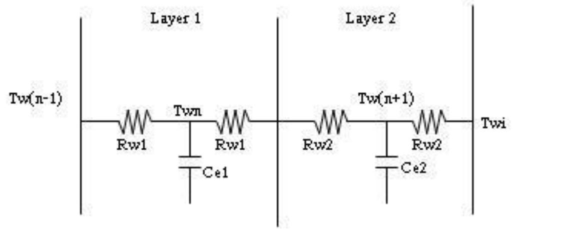

Fig. 4 Heat transfer mechanism at inner wall node

The considered point is represented as T_{wn}. When temperature of each node is not similar, conduction heat transfer occur. In dynamic heat transfer, the capacity of the wall will absorb heat energy. In this case, considered equation is expressed in energy balance equation,

$$\left(\frac{T_{w(n-1)}-T_{wn}}{R_{w1}}\right) - \left(\frac{T_{wn}-T_{w(n+1)}}{R_{w1}+R_{w2}}\right) = Ce_i\cdot\left(\frac{T_{wn}^P - T_{wn}^{P-1}}{dt}\right)$$

Eq. (15)

4.3 Internal surface node

For this node, there are 3 types of heat transfer component ,convective heat transfer, conduction heat transfer ,and radiation heat transfer. Heat transfer to air conditioning is effected by convective heat transfer ,and the change in wall temperature will effect to other walls side by radiation heat transfer. The following figure illustrates heat transfer mechanism for inside wall surface as shown in Fig. 5.

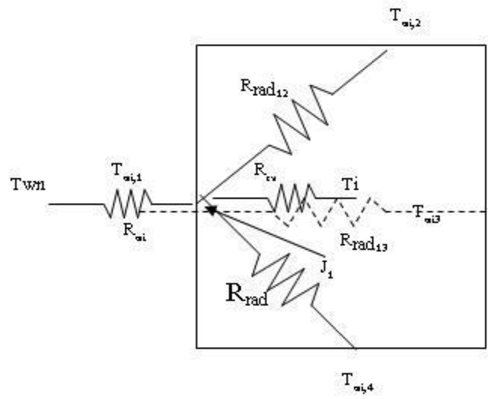

Fig. 5 Heat transfer mechanism at inside wall surface

$$\left(\frac{T_{wi}-T_i}{R_c}\right) + \frac{\varepsilon_i}{\rho_i}\cdot\left(\sigma\cdot T_{wi}^4 - J_i\right) + \sum_{j=1}^{TotalSide}\left(\frac{T_{wi}-T_{wj}}{R_{cond,i,j}}\right) + \left(\frac{T_{wi}-T_i}{R_{cond,i,air}}\right) - \alpha_i\cdot q_{sw} - \left(\frac{T_{wn}-T_{wi}}{R_{wi}}\right) = 0 \qquad \text{Eq.(16)}$$

Where the radiosity of the flux can be represented in terms of J which can be shown in eq. (17)

$$\sigma\cdot T_i^4 + \sum_{j=1}^{No.Of.Side}\left(\frac{\rho_i\cdot F_{ij}}{\varepsilon_i}\right)\cdot J_j - \left(\frac{1}{\varepsilon_i}\right)\cdot J_i = 0 \qquad \text{Eq.(17)}$$

Where

Form factor (F_{ij}) = the fraction that determine the amount of light flux leaving from diffuse area source to diffuse receiving area [6]

$$F_{1-2} = \frac{1}{\pi A_1}\iint\left(\frac{\cos\theta\cos\xi}{D^2}\right)dA_1 dA_2 \quad \text{Eq. (18)}$$

4.4 Heat transfer through glass

One major factor due to heat gain into the closed rectangular enclosure is the top glassing material.

The calculation of heat transfer through glassing can be separated into 2 components, short wave and long wave radiation. For long wave radiation, we can treat window as a wall. Heat transfer through window we can calculate it in the similar patterns as opaque wall. Short wave radiation, the heat gain coming into room is depend upon solar radiation(I_t) and transmission of window(τ) as shown in following equation,

Short wave Radiation heat gain = $\tau \cdot I_t$ Eq.(19)

Total heat gain from window = short wave radiation + long wave radiation Eq. (20)

4.5 Methodology for Temperature calculation

The heat gain calculation equations as shown in Eq. (13)- (20) are non-linear equations which can be solved by using newton raphson method with multiple equations and unknowns. The calculation methodology can be identified into 5 steps as following,

Step 1: Rewrite the equation in terms of one side of equality sign,

For external Node

$$f_1(T_{w0}, T_{wn}) =_1 \alpha_{ab}\cdot I_t + FF_{sur-W}\cdot\varepsilon\cdot\sigma\cdot(T_{surr}^4 - T_{wo}^4) + FF_{grd-W}\cdot\varepsilon\cdot\sigma\cdot(T_{grd}^4 - T_{wo}^4) + h_{co}\cdot(T_o - T_{wo}) - \left[\frac{T_{wo} - T_{wn}}{R_w}\right] = 0$$

Eq.(21)

For Internal Node

$$f_2(T_{w(n-1)}, T_{wn}, T_{w(n+1)}) = \left(\frac{T_{w(n-1)} - T_{wn}}{R_{w1}}\right) - \left(\frac{T_{wn} - T_{w(n+1)}}{R_{w1} + R_{w2}}\right) - Ce_i\cdot\left(\frac{T_{wn}^P - T_{wn}^{P-1}}{dt}\right) = 0$$

Eq. (22)

For Inside wall node

$$f_3(T_{wn}, T_{wi,1}, T_{wi,2}, ..., T_{wi,n}) = \left(\frac{T_{wi} - T_i}{R_c}\right) + \sum_{j=1}^{no.of\ side} F_{ij}\cdot q_{r(i,j)} - \alpha_i\cdot\frac{E_i}{efficacy} - \left(\frac{T_{wn} - T_{wi}}{R_{wi}}\right) = 0$$

Eq. (23)

Step 2: Assume temporary values for the variables (T and ΔT), for example T = 25 , ΔT = 1

Step 3: Calculate the values of f1,f2,and f3 at the temporary values of assumed temperature(T=25)

Step 4: Compute the partial derivatives of all functions with respect to all variables, by using assumed values from step 2 , for example,

$$\frac{\Delta f(T_1, T_2, T_3)}{\Delta T_1} = \frac{f(T_1 + \Delta T_1, T_2, T_3) - f(T_1, T_2, T_3)}{\Delta T_1} \quad \text{Eq.(24)}$$

Step 5: Setup the matrix in same form of the taylor series expansion format

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} \cdot \begin{bmatrix} x_{1,t} - x_{1,c} \\ x_{2,t} - x_{2,c} \\ x_{3,t} - x_{3,c} \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} \quad \text{Eq. (25)}$$

$$
\begin{bmatrix}
a_{110} & 0 & 0 & 0 & a_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{210} & 0 & 0 & 0 & a_{211} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{310} & 0 & 0 & 0 & a_{311} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{410} & 0 & 0 & 0 & a_{411} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{120} & 0 & 0 & 0 & a_{121} & 0 & 0 & 0 & a_{12i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_{220} & 0 & 0 & 0 & a_{221} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & a_{320} & 0 & 0 & 0 & a_{321} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & a_{420} & 0 & 0 & 0 & a_{421} & 0 & 0 & 0 & a_{42i} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & a_{131} & 0 & 0 & 0 & a_{13i} & a_{132} & a_{133} & a_{134} & a_{14i} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{231} & a_{23i} & a_{233} & a_{234} & 0 & a_{24i} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{331} & a_{332} & a_{33i} & a_{334} & 0 & 0 & a_{34i} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{431} & a_{431} & a_{432} & a_{433} & a_{43i} & 0 & 0 & 0 & a_{44i} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{151} & a_{152} & a_{153} & a_{154} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{251} & a_{252} & a_{253} & a_{254} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{351} & a_{352} & a_{353} & a_{354} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_{451} & a_{252} & a_{453} & a_{454}
\end{bmatrix}
\cdot
\begin{bmatrix}
\Delta T_{1,wo} \\ \Delta T_{2,wo} \\ \Delta T_{3,wo} \\ \Delta T_{4,wo} \\ \Delta T_{1,wn} \\ \Delta T_{2,wn} \\ \Delta T_{3,wn} \\ \Delta T_{4,wn} \\ \Delta T_{1,wn} \\ \Delta T_{2,wn} \\ \Delta T_{3,wn} \\ \Delta T_{4,wn} \\ \Delta J_1 \\ \Delta J_2 \\ \Delta J_3 \\ \Delta J_4
\end{bmatrix}
=
\begin{bmatrix}
f_{1,wo} \\ f_{2,wo} \\ f_{3,wo} \\ f_{4,wo} \\ f_{1,wn} \\ f_{2,wn} \\ f_{3,wn} \\ f_{4,wn} \\ f_{1,wi} \\ f_{2,wi} \\ f_{3,wi} \\ f_{4,wi} \\ f_{1,j} \\ f_{2,j} \\ f_{3,j} \\ f_{4,j}
\end{bmatrix}
$$

Eq. (26)

Where $a_{i,j,k}$: i = side i^{th} , j = equation j^{th}, where j=1 use eq. (21) ; j=2 use eq. (22) and j=3 use eq. (23) ; k = layer k^{th}

for example,

$$a_{110} = \frac{\Delta f_1(T_{1,wo}, T_{1,wn})}{\Delta T_{1,wo}} = \frac{f(T_{1,wo} + \Delta T_{1,wo}, T_{1,wn}) - f(T_{1,wo}, T_{1,wn})}{\Delta T_{1,wo}}$$

5. The Result Verification

In order to verify the result with experiment with computer program , the Pearson Correlation (Rxy) [10] is applied as shown in eq. (27)

$$r_{xy} = \frac{N\sum XY - \sum X \sum Y}{\sqrt{\left(N\sum X^2 - \left(\sum X\right)^2\right)\cdot\left(N\sum Y^2 - \left(\sum Y\right)^2\right)}} \quad \text{Eq.(27)}$$

Correlation coefficient (R^2) is also applied to verify the correction of the calculation and experiment data.

5.1 Result

In order to compare temperature profile between the calculation (from developed model) and experiment, all measuring data from data logger have been transferred to the computer. The temperature comparison between calculation and experiment have been done at thermocouple installed position in order to compare the temperature profile as shown in fig. 6– 11

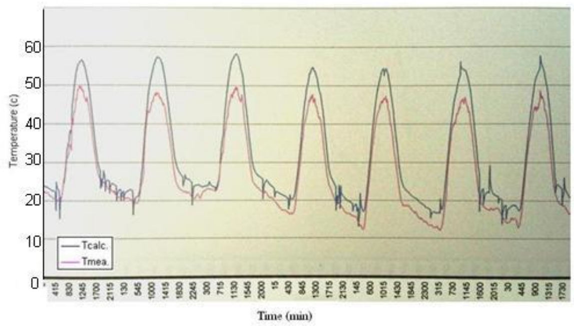

Fig. 6 The temperature profile of Glass top

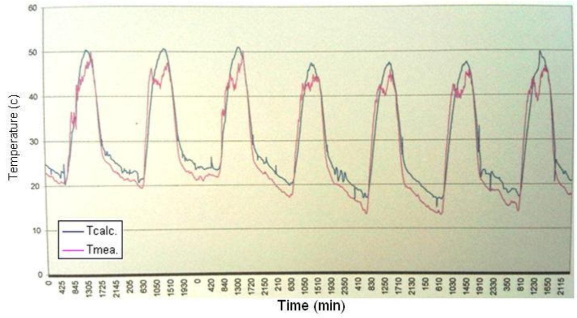

Fig. 7 The temperature profile of East wall

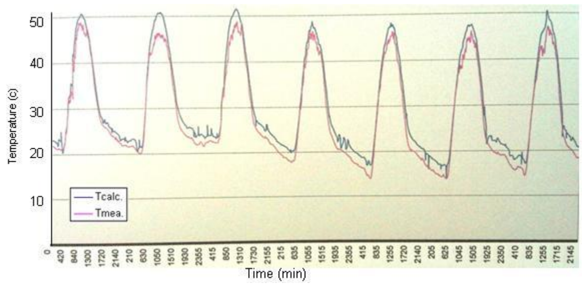

Fig. 8 The temperature profile of south inner wall

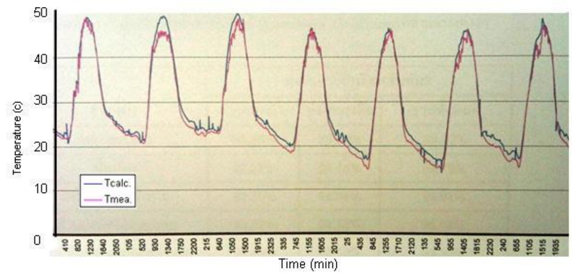

Fig. 9 The temperature profile of south ext. wall

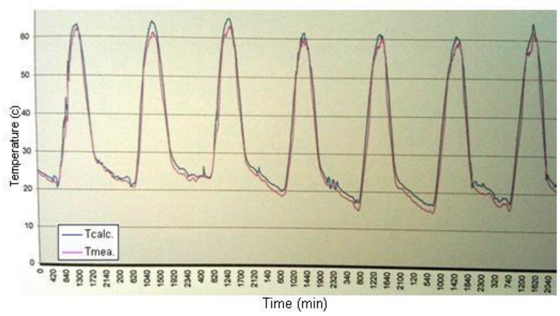

Fig. 10 The temperature profile of ground floor

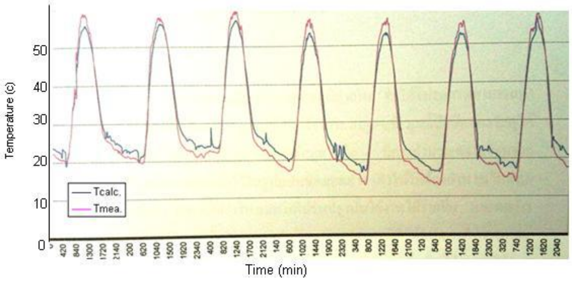

Fig. 11 The temperature profile of inside air

In order to compare the results, the statistical comparison has been applied by using pearson correlation and correlation coefficient ,eq. (27) to verify the reliability between developed model and actual experiment data. As shown in Fig. 6-11 , at day time the temperature calculation from developed model is closed to experiment than night time. Therefore, in order to compare the result, the data have been separated into 2 sets as daytime and night time. The result verification is shown in Table 1.

Table 1. Correlation coefficient (R^2) at different wall position

Position	**Day time**	**Night Time**	**Average**
Glass Cover	98.65%	78.79%	88.72%
Ground Floor	99.27%	98.14%	98.70%
inner wall south	99.30%	94.64%	96.97%
ext. wall south	99.22%	94.73%	96.98%
east wall	93.46%	93.15%	93.30%
inner air	99.41%	95.96%	97.68%

6. Conclusion and Recommendation

From the experimental results, a verification of developed mathematic model and actual result data found the reliability of 94% at day time and 89% at night time.

This is because in day time, the effect of solar radiation is a strong driven force to rise up the temperature whereas in the night time , there are no sunlight (only ambient temperature and sky temperature which is not as strong driven force as solar radiation).

This work can be further applied to solar dryer and solar cooker heat gain calculation. Even though, this developed model takes times to calculate the data (depending upon how many setup node and equation matrix size) but the results have proven to be high reliability.

7. References

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